Random regression models

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Random regression

$$
\begin{aligned}
y_{ijkl} &= \sum_{m=0}^{4} (b_{mi} x_m) \\
&+ \ldots \\
&+ h_j \\
&+ a_{jk} \\
&+ p_{ijk} \\
&+ e_{ijkl}
\end{aligned}
$$

Random regression

$$
\begin{aligned}
y_{ijkl} = & \sum_{m=0}^{4} (b_{mi} x_m) \\
+ & \ldots \\
+ & h_{Aj} + h_{Bj} t_B \\
+ & a_{Ajk} + a_{Bjk} t_B + a_{Cjk} t_C + a_{Djk} t_D \\
+ & p_{Aijk} + p_{Bijk} t_B + p_{Cijk} t_C \\
+ & e_{Aijk} + e_{Bijk} t_B + e_{Cijk} t_C \\
+ & \varepsilon_{ijkl}
\end{aligned}
$$

herd

$$\mathbf{f}'_{hj} = \begin{pmatrix} 1 & t_B \end{pmatrix}$$
$$\mathbf{f}'_{ajk} = \begin{pmatrix} 1 & t_B & t_C & t_D \end{pmatrix}$$
$$\mathbf{f}'_{pjk} = \begin{pmatrix} 1 & t_B & t_C \end{pmatrix}$$
$$\mathbf{f}'_{ejk} = \begin{pmatrix} 1 & t_B & t_C \end{pmatrix}$$

incidence matrices

$$\mathbf{Z}_h = \begin{bmatrix} \mathbf{f}_{h1}' & \mathbf{0}' & \cdots \\ \mathbf{f}_{h1}' & \mathbf{0}' & \\ \mathbf{0}' & \mathbf{f}_{h2}' & \\ \vdots & & \ddots \end{bmatrix}$$

herd

$$
\begin{aligned}
\mathbf{V} = \; & \mathbf{Z}_h \mathbf{G}_h \mathbf{Z}_h^{'} \\
+ \; & \mathbf{Z}_a \mathbf{G}_a \mathbf{Z}_a^{'} \\
+ \; & \mathbf{Z}_p \mathbf{G}_p \mathbf{Z}_p^{'} \\
+ \; & \mathbf{R}
\end{aligned}
$$

herd

$$\mathbf{G}_h = \mathbf{I} \otimes \mathbf{G}_{0h}$$

$$\mathbf{G}_{0h} = \begin{bmatrix} \sigma^2_{hA} & \sigma_{hAB} \\ \sigma_{hAB} & \sigma^2_{hB} \end{bmatrix}$$

animal

$$\mathbf{G}_a = \mathbf{A} \otimes \mathbf{G}_{0a}$$

$$\mathbf{G}_{0a} = \begin{bmatrix} \sigma^2_{aA} & \sigma_{aAB} & \sigma_{aAC} & \sigma_{aAD} \\ & \sigma^2_{aB} & \sigma_{aBC} & \sigma_{aBC} \\ & & \sigma^2_{aC} & \sigma_{aCD} \\ & & & \sigma^2_{aD} \end{bmatrix}$$

residual

$$\mathbf{R} = \mathbf{Z}_e \mathbf{G}_e \mathbf{Z}_e^{'} + var\left(\varepsilon\right)$$

$$\mathbf{G}_e = \mathbf{I} \bigotimes \mathbf{E}_0$$

$$var\left(\varepsilon\right) = \mathbf{diag}\left\{\sigma^{\mathbf{2}}_{\varepsilon\mathbf{w}}\right\}$$

Prediction of breeding value

$$EBV_{305} = \sum_{x=1}^{305} (\hat{a}_{Ajk} + \hat{a}_{Bjk} t_B + \hat{a}_{Cjk} t_C + \hat{a}_{Djk} t_D)$$

$$= \hat{a}_{Ajk} \sum_{x=1}^{305} (1) + \hat{a}_{Bjk} \sum_{x=1}^{305} (t_B) +$$

$$+ \hat{a}_{Cjk} \sum_{x=1}^{305} (t_C) + \hat{a}_{Djk} \sum_{x=1}^{305} (t_D)$$

$$\sigma^{\hat{2}}_{gx} = \mathbf{f}'_x \hat{\mathbf{G}}_{0a} \mathbf{f}_x$$

$$\hat{\sigma}_{gx1x2} = \mathbf{f}'_{x1} \hat{\mathbf{G}}_{0a} \mathbf{f}_{x2}$$

Persistancy

$$EBV_p = \sum_{x=60}^{280} (\hat{a}_{Ajk} + \hat{a}_{Bjk}t_B + \hat{a}_{Cjk}t_C + \hat{a}_{Djk}t_D)$$

$$= \hat{a}_{Ajk} \sum_{x=60}^{280} (1) + \hat{a}_{Bjk} \sum_{x=60}^{280} (t_B) +$$

$$+\hat{a}_{Cjk} \sum_{x=60}^{280} (t_C) + \hat{a}_{Djk} \sum_{x=60}^{280} (t_D)$$

$$EBV_x = (\hat{a}_{Ajk} + \hat{a}_{Bjk}t_B + \hat{a}_{Cjk}t_C + \hat{a}_{Djk}t_D)$$

$$\hat{\mathbf{a}}_{jk}^{'} = \left(\begin{array}{cccc} \hat{a}_{Ajk} & \hat{a}_{Bjk} & \hat{a}_{Cjk} & \hat{a}_{Djk} \end{array} \right)$$

animal

Model order	LG[1] EV	LG[1] %	LG[2] EV	LG[2] %	LG[3] EV	LG[3] %	LG[4] EV	LG[4] %
0th	36.19	96.70	37.48	90.93	38.23	89.64	38.59	89.09
1st	1.23	3.30	3.20	7.76	2.97	6.97	2.89	6.67
2nd	-		0.54	1.31	1.14	2.28	1.37	3.16
3rd	-		-		0.30	0.71	0.29	0.67
4th	-		-		-		0.17	0.41
	37.42		41.22		42.64		43.31	

permanent environment

Model	LG[1]		LG[2]		LG[3]		LG[4]	
order	EV	%	EV	%	EV	%	EV	%
0th	42.66	97.53	42.35	91.68	43.52	91.89	44.05	91.11
1st	1.08	2.47	3.40	7.36	3.30	6.96	3.54	7.33
2nd	-		0.44	0.96	0.52	1.10	0.59	1.21
3rd	-		-		0.03	0.06	0.17	0.35
4th	-		-		-		0.00	0.00
	43.74		46.19		47.37		48.35	